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Stream meanders on a smooth hydrophobic surface

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This paper reports an experimental study of a meandering water stream upon an inclined smooth hydrophobic surface. It was found that the sinuosity (ratio of the total stream length to the length of the projection of the stream on the line of maximum slope of the surface) increases with both increasing discharge rate and surface slope. It was observed that the meandering pattern is not always stable: once the discharge rate exceeds the upper critical value, the meandering pattern becomes unstable, whereas, when the discharge rate is smaller than the lower critical value, the water stream becomes discontinuous, and normally forms droplets, sliding successively down the sloping surface. It was found that, with increasing surface slope, the upper critical value decreases exponentially, while the lower critical value decreases only gradually. It was found that, when a system of stable meanders is formed on the surface, the meander loops are smoothly curved, swinging gradually from left-handed to right-handed deflections from the line of fastest descent, and *vice versa*, with an almost constant amplitude and wavelength. The stable meandering pattern migrates gradually down the sloping surface.

The observations showed that the central axis of the meandering stream does not coincide with the locus of the highest points of the stream, the highest points being displaced towards the outside of each bend: the cross-sectional profile of the stream is thus usually asymmetrical. It was found that the cross-sectional area of the stream varies cyclically, with one increase and one decrease associated with each bend of the stream. This cyclic variation is repeated many times along the length of the stream, with each point of maximum cross-sectional area located close to a bend. A secondary reversing spiral flow was observed in the stream, and it was found that the sense of rotation of the flow is reversed at each bend.

A plausible mechanism of these stream meanders is proposed on the basis of the present results, involving the existence of hysteresis of the contact angle between water and Plexiglas, the presence of asymmetrical surface-tension forces on the stream, and the acceleration and deceleration of the stream as it swings from loop to loop.

1. Introduction

It is an everyday observation that streams of rain on window panes tend to meander. The scientific investigation of this phenomenon, however, appears to date only from 1960, when Tanner investigated streams running down the underside of a nearly horizontal glass plate. The situation has since been considered by Gorycki (1973), Nakagawa (1982) and others.

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It is clear that the motion of the meandering stream on an inclined surface is driven by gravity, and that the shear stress at the liquid–solid interface is much greater than that at the liquid–air interface. It is also apparent that surface tension should play a major role in determining the motion.

Another factor likely to be important is the 'hysteresis' of the angle of contact between the water and the solid surface, the contact angle being known to be considerably greater for an advancing water contact line than for a receding contact line. It is because of the existence of this contact-angle hysteresis that small rain droplets can stick to a vertical window pane without running downwards. Without any hysteresis a droplet of any size would continue to move on any but a perfectly horizontal surface.

Experiments were performed in which water was released on an inclined smooth hydrophobic surface, and the resulting meandering stream was observed using measurements of the stream cross-section and the sinuosity (the ratio of the total stream length to the length of the projection of the stream on the line of maximum slope of the surface), together with dye-streak visualization of the motion.

2. Experiments

Figure 1 shows a schematic diagram of the experimental arrangement. The optically smooth Plexiglas plate was 100 cm long and 60 cm wide, and the water tank allowed a constant discharge rate of water through a vinyl tube, of 10 mm inner diameter. The tube mouth was arranged on the centreline of the plate, at a point 20 cm down the slope from the upper edge. The experimental variables were the discharge rate, measured by weighing the water discharged in a given time, and the surface slope, measured with a protractor. The maximum discharge rate for each surface slope is an amount above which the meandering pattern becomes unstable, and the surface slope was varied from 5° to 85° in 5° steps.

The cross-section of the stream was measured pointwise, using a slender needle, 0.1 mm thick, held perpendicular to the surface slope. Height measurements were made at 5 mm intervals along the direction of maximum slope, and at 1 mm intervals along a line normal to the central axis of the meandering stream. The total length of the stream was determined by tracing the plan locus of its central axis, using ink, on to the transparent plate. The Reynolds number of the flow within the vinyl pipe was in the range 100–650, well below the critical value, and the flow was observed to be essentially laminar.

The flow within the stream was visualized using a dye streak of methylene blue aqueous solution, released into the flow 10 mm downstream from the tube mouth via a hypodermic needle (0.3 mm diameter). Although this material is surface-active, it was apparent in this work that the visible streak remained well within the meandering stream, near the central core, and it is felt that the dye had no effect on the surface tension of the water in the part of the stream under consideration here.

During the course of the experiment it was found that the motion of the stream is very sensitive to the condition of the plate surface. For example, the stream would tend to follow parts that had already been wetted. Thus a dry surface was carefully prepared for each experimental run, wiping the surface with soft hygroscopic tissues, and then leaving the surface for 30 minutes prior to the next run.

In the interpretation of the experimental results the right or left sides of the stream are defined facing the direction of flow.

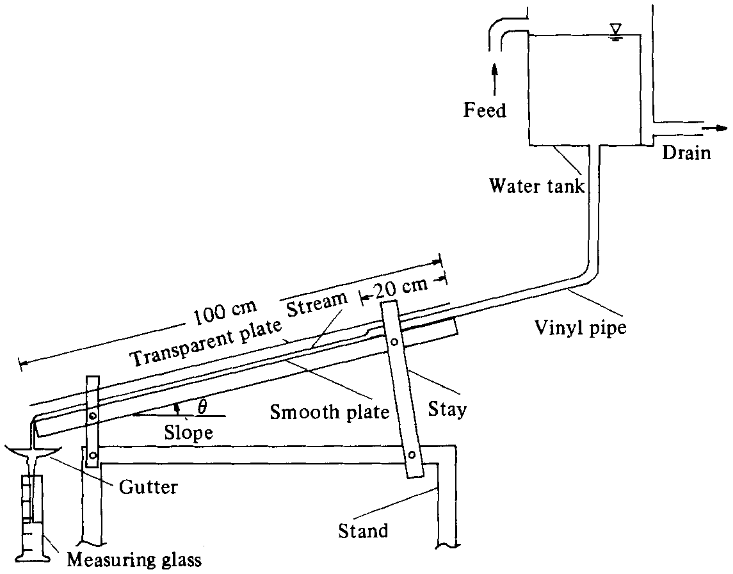

FIGURE 1. Schematic diagram of the experiment.

3. Results

Figure 2 shows a stable meandering stream system on the smooth surface, with grid lines drawn on the underside of the plate. The meander loops are invariably smoothly curved, swinging gradually from left-handed to right-handed deflection, and *vice versa*, with an almost-constant amplitude and wavelength of the meanders.

Figure 3 shows how the sinuosity of the stream meanders depends on the discharge rate and the surface slope respectively. When the surface slope is constant, the sinuosity increases with increasing discharge rate. Furthermore, the rate of increase of the sinuosity increases with increasing surface slope until the surface slope becomes 35°, above which the sinuosity is almost independent of the surface slope. For a constant discharge rate, the sinuosity increases with increasing surface slope.

It was observed that the meandering stream is not always stable in pattern, but that, once the discharge exceeds a critical value, the upper critical value, the stream course begins to change, and the unstable portion at the downstream end gradually increases with further increment of the discharge rate. On the other hand, when the discharge rate becomes smaller than the lower critical value, the water stream becomes discontinuous and normally forms droplets, sliding successively down the sloping surface. It should be noted that, in the experiments reported here, no perfectly straight stream was observed for any condition of discharge rate or surface slope, any stable stream meandering to some degree.

Figure 4 shows a diagram defining the regions of the observed stream patterns, viz the unstable meandering stream, the stable meandering stream, and the discontinuous droplets, the abscissa and ordinate being surface slope and discharge rate respectively. It is apparent from figure 4 that, as the surface slope increases, the upper critical value of discharge rate decreases exponentially, whereas the lower critical value, defined as the discharge rate below which the water stream forms clearly discontinuous droplets, decreases only gradually. It is, however, important to note that, when the surface slope is smaller than about 30°, no lower critical value is observed: in this

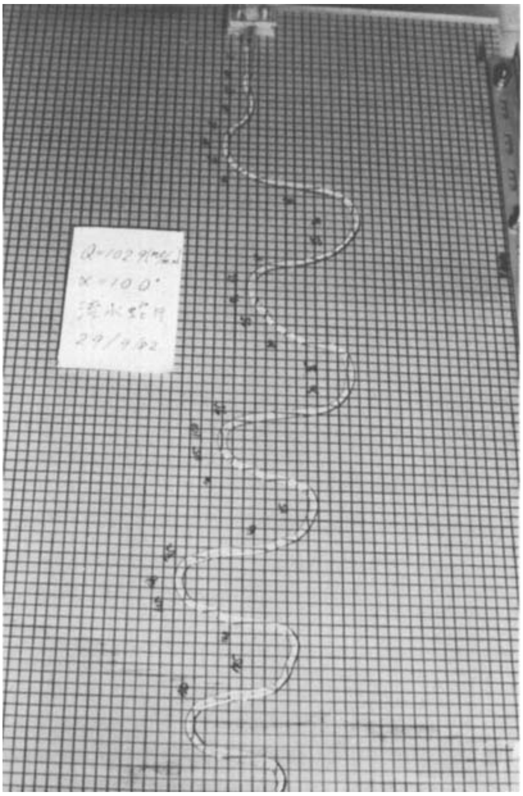

FIGURE 2. A stable meandering stream on the smooth hydrophobic Plexiglas surface. Discharge = 1.72 cm^3/s; surface slope = 10°. The flow direction is from top to bottom of the figure.

region of surface slope discontinuous droplets are not formed, but a stable meandering stream is formed for all values of the discharge rate.

Figure 5 shows the upstream part of the stable meandering stream shown in figure 2. The plan locus of the highest points of the stream surface, and the measured cross-sections at 5 mm intervals along the maximum slope direction, are also shown in this figure. The locus of the highest points is shown as a dotted line, with the central axis of the stream indicated by a dash–dotted line. It is clear that the two lines do not coincide, and that the highest points of the stream tend to lie on the outside of the meander bends. Between $X = 0$ cm and $X = 7$ cm the cross-section is almost symmetrical, but the asymmetry becomes marked as soon as the meandering begins. Between each two bends the two loci cross each other, and the cross-section is once more almost symmetrical.

Figure 6 shows how the stream height, the stream width and the cross-sectional area vary along the stream, as a function of the distance X down the line of maximum slope. Figure 6 indicates that, while the cross-sectional area fluctuates in the $X = 0$ cm to $X = 7$ cm region, the variation is much more specific further downstream, where the meandering is fully developed. The decrease/increase cycle of the cross-sectional area is repeated with every bend of the stream.

It should be noted that, for each of the bends shown in figure 5, the cross-sectional area is greatest shortly before a bend apex, and there is a region of smaller cross-sectional area at the apex itself. The variations of the maximum stream height

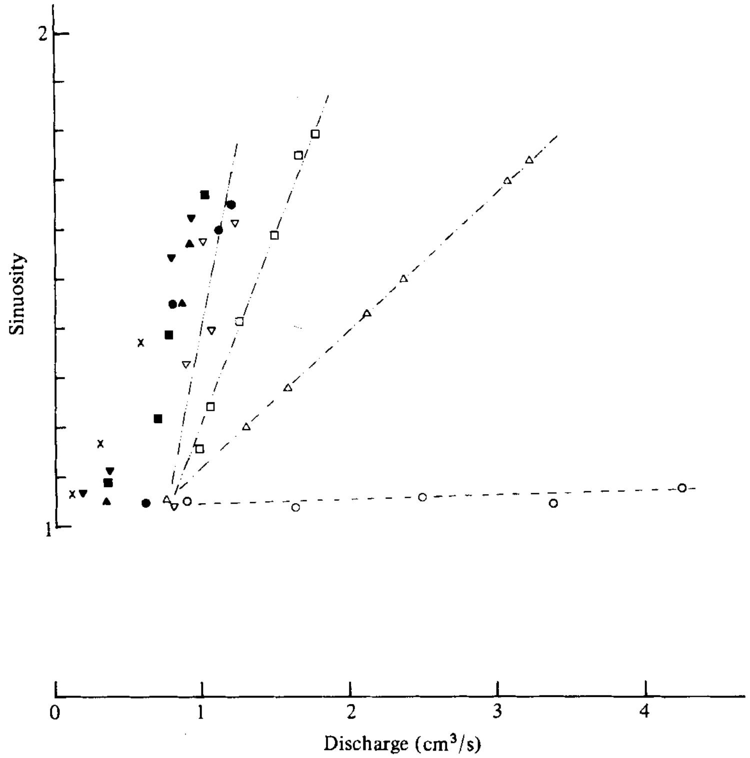

FIGURE 3. Sinuosity *vs.* discharge for each surface slope: ○, $\alpha = 5°$; △, 15°; □, 25°; ▽, 35°; ●, 45°; ▲, 55°; ■, 65°; ▼, 75°; ×, 85°.

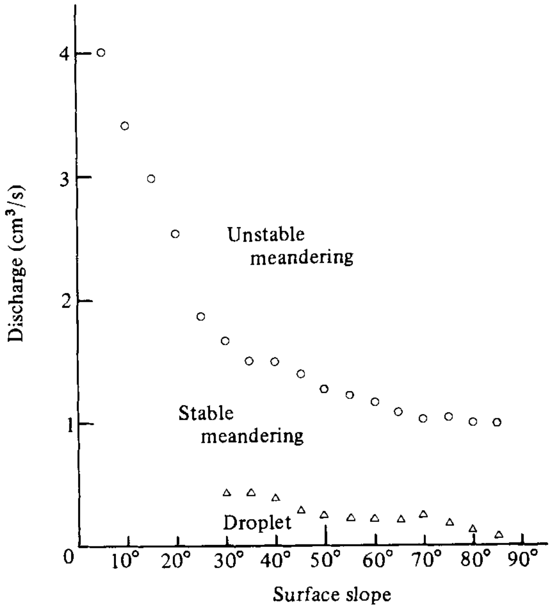

FIGURE 4. Diagram defining regions of the observed stream patterns.

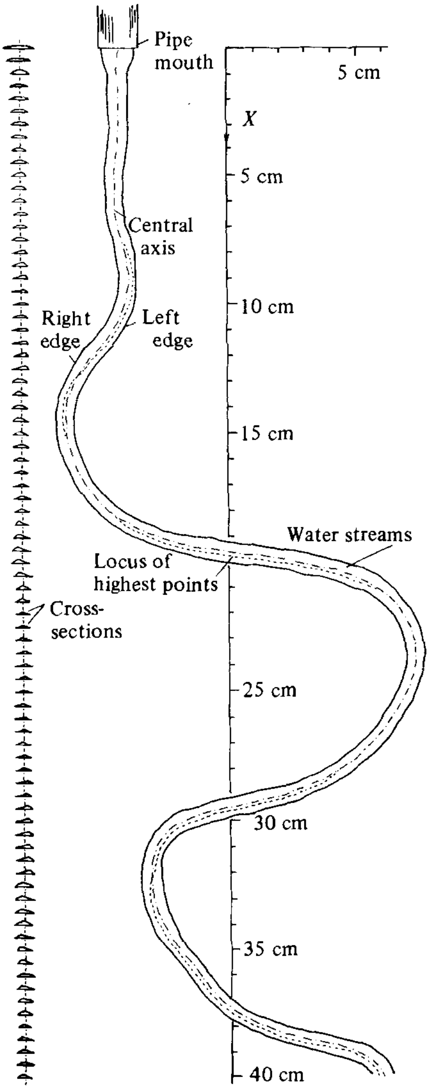

FIGURE 5. Plan locus of the highest points of the stream surface, and the stream profiles at 5 mm intervals along the direction of maximum slope. The conditions are the same as in figure 2.

and the stream width are similar to the cross-sectional area variation. It is important to note that, since the volume flow in the stream must be constant, the points of maximum cross-section must be points of minimum mean velocity of the stream.

Figure 7 shows the visualization of the spiral secondary flows, using blue dye released into the stream at the top of the photograph. The dye release point is near the centre of the stream, close to the surface of the plate.

The dye filament flows initially on the right-hand side of the stream, and then moves to the left-hand side as the stream swings right. At a point near the end of the straight portion of the right-moving stream the dye streak bifurcates, the branch that stays close to the plate surface continuing its course on the left of the stream and the branch nearer the stream surface crossing over to the right-hand side. This indicates a right-handed spiral secondary flow along the main flow direction. However, in-between

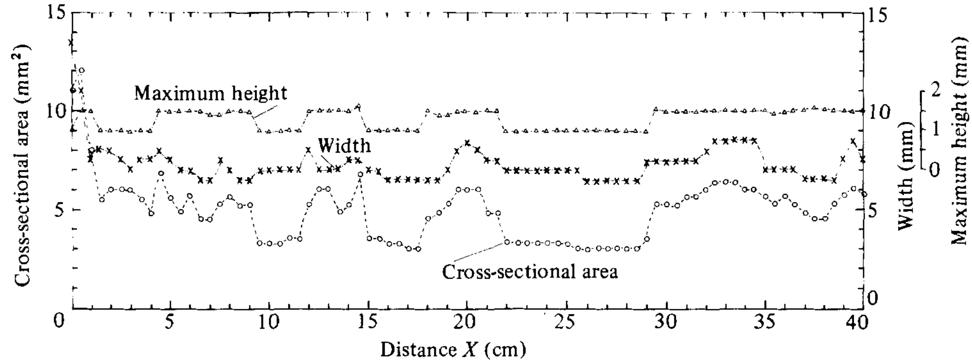

FIGURE 6. Variation of the cross-sectional area, stream width and maximum height of the stream, on the distance along the direction of maximum slope.

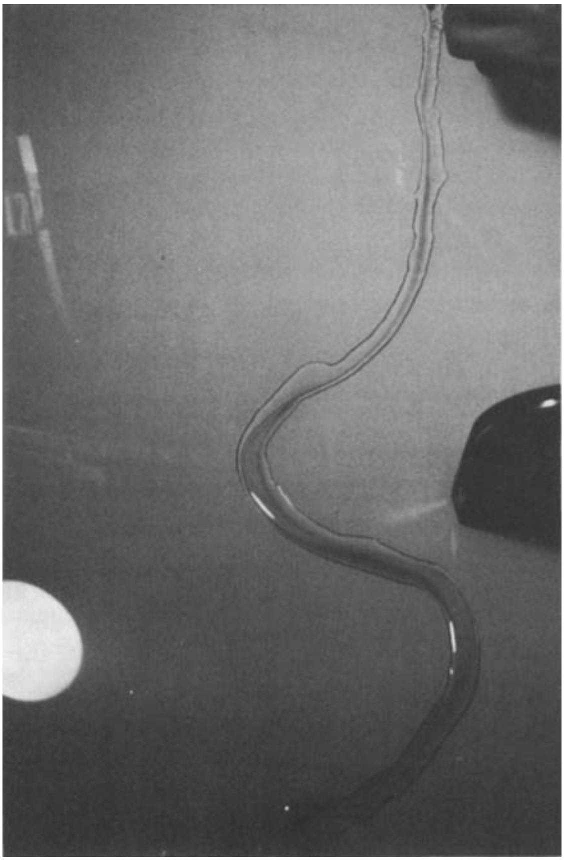

FIGURE 7. Magnified view of the spiral secondary flows at the stream bends. Discharge = 2.67 cm^3/s, surface slope = 10°. The flow direction is from top to bottom of the figure.

the first and second bends the branch closer to the plate crosses the central axis to the right-hand side, and the other (near-surface) branch crosses back to the left-hand side. Thus on the approach to the second bend the spiral flow is completely reversed, i.e. left-handed.

The diffusion of the dye throughout the stream further down the plate made it impossible to identify clearly the sense of rotation, but it was apparent that the sense of the rotation changed completely at each subsequent bend.

4. A proposed mechanism of stream meanders on a smooth hydrophobic surface

It is possible, using the results obtained in the present investigation, to infer plausible mechanisms for the stream-meander phenomenon. It is proposed that two principal mechanisms are involved, relating to the originating instability, and relating to the side-to-side oscillation of the stream, respectively.

4.1. *The origin of the initial instability*

An initially symmetrical water stream beginning its fall, under gravity, down an inclined perfectly smooth plane might be expected to accelerate until the force exerted on the stream by the shear stress in the boundary layer on the solid surface became equal to the gravitational force. The only change in the system expected in this case would be a flattening of the cross-sectional profile, and an accompanying reduction of the cross-sectional area – both associated with the acceleration of the stream under gravity. It is reasonable to assume that any subsequent increases or decreases of the cross-sectional area should be indications of decreases or increases (respectively) in the mean stream velocity.

In this idealized symmetrical case, the width of the stream at any point along its length – the cross-stream distance between the two confining contact lines – would be determined by the cross-sectional area of the stream at that point, and by the contact angle of the water on the solid surface. Initially, when the water is discharged on to the surface, the width might also be related to the exit diameter of the tube. At this stage the system becomes influenced by the phenomenon of contact-angle hysteresis.

It is well known that the contact angle of water on a solid surface is not a constant, except in some unusual cases of special prepared surfaces, of highly pure materials (Blake & Haynes 1973). With most practical materials, including Plexiglas or Perspex (Stepanov, Volyak & Tarlakov 1977), it is found that the contact angle can vary – considerably – between two (more or less constant) limits, represented by the cases of advancing and receding contact lines measured as the water is encouraged to spread or to recede on the solid surface. Stepanov *et al.* report that, for water on Plexiglas, the advancing and receding contact angles are 63° and 32°, respectively. As the water contact line changes from the advancing condition to the receding condition the contact line is observed to 'stick' in the same place, and the contact angle changes smoothly between the two limits.

As the stream accelerates down the slope, the cross-sectional area would decrease, and the contact angle would then tend to decrease towards the receding contact-angle value. When the angle actually reached this lower limit, the contact line would recede, and the stream width would decrease. This effect may be seen clearly in the first three profile cross-sections shown in figure 5.

On the basis of this 'first-guess' idealized picture there is no obvious physical reason

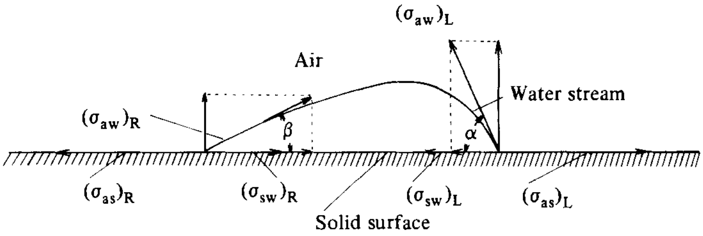

FIGURE 8. The surface-tension forces acting on the water stream when the profile is asymmetrical. The σ-values represent the surface tensions of the relevant surfaces, the suffixes aw, as and sw standing for air–water, air–solid and solid–water respectively, and the letters L and R indicating the relevant sides of the stream, left and right.

why the stream should at any stage diverge to one side or the other. If, however, the surface profile should become (for any reason) asymmetrical, then the result is certain to be an imbalance in the forces acting on the stream, and it appears that this imbalance is often sufficient to cause a major instability of the straight-line motion of the stream. Let us assume, for the moment, that there is a reasonable explanation for the origin of the asymmetry, and examine the consequences.

The principal factor in the appearance of an asymmetrical stress on the stream is likely to come from the asymmetrical profile itself. The surface-tension forces that act on the stream always act tangentially to the water surface, and the horizontal component of the surface-tension force acting on the stream is the product of the surface tension and the cosine of the contact angle, as shown in figure 8. In considering the resultant effect of the contact lines at the two sides of the stream, therefore, it may be seen that there will be a net across-the-stream surface-tension force, acting in the direction of the thinner part of the stream – towards the side with the smaller contact angle. The effect of the asymmetry may thus be to swing the stream in the direction of its thinner side. Once the stream has begun to swing to one side a second surface-tension force will come into play, associated with the curvature of the whole stream, which will pull the stream centripetally in the direction of the curvature.

In our search for the cause of the initial asymmetry that leads to the above behaviour we need look no further than the obvious variations in the water/Plexiglas contact angle that are illustrated in figure 7. The hysteresis effect described above – variations in the contact angle at any particular point as the water is encouraged to advance or recede over the Plexiglas surface – is closely associated with a significant spatial variation of the effective contact angle from point to point on the surface. In Plexiglas it is probable that the hysteresis itself is the result of microscopic variations of molecular composition (or polymer cross-linking) in the surface layer, and it is clear that macroscopic fluctuations of a similar nature also occur, which give rise to such spatial variations. The irregularities of the contact lines shown clearly in figure 7, and to a lesser extent also in figure 5, are indications that the two contact angles differ considerably in these regions. Residual hydrodynamic turbulence in the predominantly laminar stream could also have an effect in promoting asymmetries.

4.2. *The mechanism that maintains the meandering motion*

Once the stream has been deflected sideways by the above mechanism, it is eventually constrained to travel along a more or less straight, gently inclined path by the asymmetry of the profile, now accentuated by the sideways action of gravity on the stream. As the contact angle on the lower side of the stream increases to the advancing limit the stream will tend to spread down the slope, and 'straighten out' the loop. The stream will then continue along its straight path towards the next bend.

The stream velocity, as was indicated above, is expected to be the result of a balance between the gravitational force and the shear stress at the water/solid interface. The acceleration of the stream in the first few centimetres of the flow down the slope has already been noted. When the stream has swung in one direction, initiated by the mechanism proposed above, and has been constrained to travel in a more or less straight path, there are several notable changes in its force balance compared with its original 'vertical' flow. First, the asymmetrical surface-tension force associated with the asymmetrical profile is now (at least to some extent) balanced by the gravitational force, with the thicker region of the profile naturally taking up a position lower than the thinner region.

Secondly, the component of the gravitational force that tends to accelerate the stream in its chosen direction is now very much reduced, and, assuming that the gravity/shear-stress balance was earlier being approached, it is likely that the shear stress would subsequently predominate in a region where the stream is moving to one side in a loop. Evidence for this view is apparent in figure 6, where it may be seen that the cross-sectional area of the stream tends to increase along these regions, indicating a decelerating stream velocity.

As the stream decelerates and the cross-sectional area increases, the profile tends to steepen on its lower side, and eventually the stream must advance down the slope once more. As this happens, the sideways momentum of the stream carries the 'bulge' in the profile to the outside of the bend once more, and the stream is set to continue its swing in the opposite direction, subject to the same asymmetry forces it experienced at the beginning of the first bend. Momentum is regained as the stream moves, more vertically, around the bend, and the stream is thus set up for another straight section in the opposite direction.

At each bend, the outwards motion of the bulge in the profile – towards the outside edge of the stream – appears in the form of a spiral flow of limited extent, and the swinging of the bulge from left to right and *vice versa* – always moving to stay on the outside of the bends – is seen as an apparent periodic reversal of the spiral flow.

Such rotary motions of liquids on hydrophobic surfaces are known in other circumstances, one of the best examples being the rolling of a liquid drop on a paraffin-wax surface. In this case the liquid layer adjacent to the solid surface appears to stick to the surface, lifting off behind the drop in caterpillar-track fashion.

The continued sinuous motion of the stream results from the repetition of the process just described: a slowing down of the sideways-moving stream, followed by a downwards movement into the bend, accompanied by a shift of the profile asymmetry and an acceleration of the stream, followed by entry into the next sideways movement, in the direction opposite to the earlier one.

It is apparent that the spiral flow observed in these experiments is the result of the meandering behaviour, and not *vice versa*, in contradiction of the hypothesis of Tanner (1960).

5. Conclusions

It was found that the sinuosity (ratio of the total stream length to the length of the projection of the stream on the line of maximum slope of the surface) increases with both increasing discharge rate and increasing surface slope. It was observed that the meandering pattern is not always stable: once the discharge rate exceeds the upper critical value, the meandering pattern becomes unstable, whereas when the discharge rate is smaller than the lower critical value the stream becomes discontinuous, forming droplets which slide successively down the sloping surface. It was found that, with increasing surface slope, the upper critical value decreases exponentially, while the lower critical value decreases only gradually.

It was found that, when a system of stable meanders is formed on the surface, the meander loops are smoothly curved, swinging gradually from left-handed to right-handed deflections from the line of fastest descent, and *vice versa*, with an almost constant amplitude and wavelength. The stable meandering pattern migrates gradually down the sloping surface.

The central axis of the stream does not coincide with the locus of highest points on the stream surface, the asymmetry of the profile changing completely as the stream swings from a right loop to a left loop (and back again) so that the bulge (the highest point in the cross-sectional profile) tends to be always on the outside of the bend. A form of reversing spiral flow is observed to be associated with this asymmetry shift. The cross-section of the stream becomes largest, and the stream velocity smallest, shortly before the loop reaches its end and reverses its travel.

It is believed that the meandering may be explained in terms of an instability to asymmetrical surface-tension forces on either side of the stream, brought about by the observed hysteresis in the water/Plexiglas contact angle. The initial disturbance needed to promote this instability appears to come from macroscopic variations in the contact angle. The meandering behaviour is determined by the relative changes in stream velocity at different points, the stream accelerating at the ends of the loops, and decelerating in the relatively straight sections between the loops.

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